

Compound Poisson:

Feller
p 221

$$S_N = X_1 + X_2 + \dots + X_N$$

N random

$$\Pr\{S_N = j\} = \sum_{n=0}^{\infty} g_n \{f_j\}^{n*}$$

$$f(s) = \sum_{i=0}^{\infty} f_j s^i$$

$$\Pr\{X_k = j\} = f_j$$

$$\Pr\{N = n\} = g_n$$

$$g(s) = \sum_{n=0}^{\infty} g_n s^n$$

$$\sum g_n f(s)^n$$

(Zu beweisen ist
der Gen. funktion
~~for fixed n~~
für jedes fixe n)

$$= g(f(s))$$

Example.

(i) $f(s) = q + ps$ $g(s) = e^{-\lambda + \lambda s}$

$$g(f(s)) = e^{-\lambda q + \lambda p s} = e^{-\lambda p(1-s)}$$

(ii) $g_n = (1-\alpha)r^n$ $g(s) = \frac{1-\alpha}{1-\alpha s}$

$$g(f(s)) = \frac{1-\alpha}{1-\alpha q - \alpha p s} = \frac{1-\alpha}{\alpha(1-s)}$$

$$= \frac{1-\alpha}{1-\alpha s} \quad \alpha = \alpha p / (1-\alpha q)$$