Yule distribution (Simon):

$$B_{(i,\,\rho+1)} = \frac{\Gamma_i \, \Gamma_{(\rho+1)}}{\Gamma_{(i+\rho+1)}} \qquad 0<i \quad 0<\rho<\infty$$

$$= \int_0^1 x^{i-1}(1-x)^\rho \, dx$$

$$\frac{\Gamma_i}{\Gamma_{i+k}} \sim i^{-k}$$

Maximum Likelihood

Scoring $\boxed{\theta - \theta_0 = \delta\theta}$ Rao p.165

$$\frac{\partial \log L}{\partial \theta} \simeq \frac{\partial \log L}{\partial \theta_0} + \delta\theta \frac{\partial^2 \log L}{\partial \theta_0^2}$$

$$\frac{\partial \log L}{\partial \theta_0} - \delta\theta \, I_{(\theta_0)} = 0$$

Grouped Data:

$$\log L = f_1 \log \pi_1 + \cdots f_k \log \pi_k$$

Score at θ: $\frac{\partial \log L}{\partial \theta} = \frac{f_1}{\pi_1}\frac{\partial \pi_1}{\partial \theta} \cdots \frac{f_k}{\pi_k}\frac{\partial \pi_k}{\partial \theta}$

$$I(\theta) = f \sum_{i=1}^{k} \frac{1}{\pi_i}\left(\frac{\partial \pi_i}{\partial \theta}\right)^2 \qquad f = f_1 + \cdots f_k$$

$\frac{1}{\pi_i}\frac{\partial \pi_i}{\partial \theta}$ Score

$\frac{1}{\pi_i}\left(\frac{\partial \pi_i}{\partial \theta}\right)^2$ Information

supplied by the ith class

to be calculated

first thing