

Minimum χ^2 .

$$\chi^2 = \sum_{i=1}^s \frac{1}{n_i} \sum_{j=1}^k \frac{(p_{ij} - \pi_{ij})^2}{\pi_{ij}} = \sum_i \frac{1}{n_i} \left(\sum_j \frac{p_{ij}^2}{\pi_{ij}} - 1 \right)$$

$$\frac{\partial \chi^2}{\partial \theta_i} = -\frac{1}{n_i} \sum_j \left(\frac{p_{ij}}{\pi_{ij}} \right)^2 \frac{\partial \pi_{ij}}{\partial \theta_i} = 0$$

$$\chi'^2 = \sum_i \frac{1}{n_i} \sum_j \frac{(p_{ij} - \pi_{ij})^2}{p_{ij}} = \sum_i \frac{1}{n_i} \left(\sum_j \frac{\pi_{ij}^2}{p_{ij}} - 1 \right)$$

$$\frac{\partial \chi'^2}{\partial \theta_i} = 2 \sum \frac{1}{n_i} \sum \frac{\pi_{ij}}{p_{ij}} \frac{\partial \pi_{ij}}{\partial \theta_i} = 0$$
