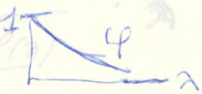
In Laplace Tr. (p. 441)

$$V(t) = G(t) + \int_0^t V(t-x)\,F\{dx\}$$

$$v(t) = g(t) + \int_0^t v(t-x)\,F\{dx\}$$

$$\psi(\lambda) = \frac{\gamma(\lambda)}{1-\varphi(\lambda)}$$

a) F proper, $\varphi(0) = 1$ $\quad \varphi(\lambda) < 1 \quad -\mu \to 0$



$$\omega = \frac{1}{1-\varphi} = \sum_0^\infty \varphi^n$$

$$\psi = \omega\gamma$$

$$V(t) = \int_0^t G(t-x)\,U\{dx\}$$

Asympt.: $\quad G(\infty) < \infty$, F has exp. μ

$$V(t) \sim \mu^{-1} G(\infty)\,t \qquad t \to \infty$$

$$V(t+h) - V(t) \to \mu^{-1} G(\infty)\,h$$

b) $F(\infty) < 1$, $\quad G(\infty) < \infty$

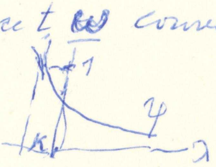
$\varphi(0) = F(\infty) < 1 \qquad \omega(0) < \infty$

V is bounded

c) $F(\infty) > 1$ $\quad$ assume $\varphi(\lambda) \to 0$ as $\lambda \to \infty$ (no atom at origin)

$\varphi(\kappa) = 1$ $\quad$ root $\kappa > 0$

argument under a) applies for $\lambda > \kappa$ (Laplace t. ω converges only for $\lambda > \kappa$)



$V = e^{\kappa x} V^{\#} \qquad \psi^{\#}(\lambda) = \psi(\lambda+\kappa)$

$e^{-\kappa x} F = F^{\#}$

$e^{-\kappa x} G = G^{\#}$