Theorem: there exist only two types of random walks;

① Oscillating with prob. 1 between $-\infty$ and ∞

$$E(\mathcal{T}_1) = \infty, \quad E(\mathcal{T}_1^-) = \infty$$

Both ascending and descending processes are persistent

② Drift to $-\infty$ (say)

ascending process terminating, descending proper

$S_n \to -\infty$, max. ≥ 0 finite,

$$P\{M \leq x\} = [1 - H(\infty)]\, \Psi(x)$$

$\boxed{\text{p. 380}}$

WALD'S IDENT.

Expectation of $\mathcal{H}_1$

if $\mathcal{H}_1$ proper, and $E(\mathcal{H}) < \infty$, then X_1 has $E(X_1) \geq 0$

$$X_n = X_n^+ - X_n^-$$

$$P\{\mathcal{H}_1 > x\} \geq P\{X_1 > x\}$$

$$E(X_1^+) \leq E(\mathcal{H}_1) < \infty$$

since $\mathcal{H}_1$ proper, with prob one, infinitely many S_n will be positive

$$n^{-1}(X_1^- + \cdots X_n^-) < n^{-1}(X_1^+ + \cdots X_n^+)$$

for infinitely many n

bounded by the strong law of large numbers)

$$\downarrow$$

$$E(X_1^-) < E(X_1^+)$$