1. Ähnlichkeitssatz

$$\mathcal{L}f(at) = \int_0^\infty f(at)e^{-pt}dt$$

$$at = \tau \quad dt = \frac{d\tau}{a}$$

$$\mathcal{L}f(at) = \frac{1}{a}\int_0^\infty f(\tau)\, e^{-\frac{p}{a}\tau}d\tau$$

$$\cancel{\mathcal{L}f} \quad \mathcal{L}f(at) = \frac{1}{a}\varphi\left(\frac{p}{a}\right)$$

$$\frac{1}{a}\mathcal{L}f\left(\frac{t}{a}\right) = \varphi(ap)$$

2. Ähnlichkeitssatz

Verschiebung

$$f(t) \equiv 0, \quad t < 0$$

$$\mathcal{L}f(t-a) = \int_0^\infty f(t-a)e^{-pt}dt = \int_a^\infty f(t-a)e^{-pt}dt,$$

$t - a = \tau$

$$\mathcal{L}f(t-a) = \int_0^\infty f(\tau)\, e^{-p(\tau+a)}d\tau$$

$$\mathcal{L}f(t-a) = e^{-pa}\mathcal{L}f(t)$$

$$\mathcal{L}E(t,a) = e^{-pa}\mathcal{L}(t,0)$$

$$\varphi(p+b) = \int_0^\infty f(t)\, e^{-(p+b)t}dt = \int_0^\infty e^{-pt}\left\{f(t)\, e^{-bt}\right\}dt$$

$$\mathcal{L}\left\{f(t)\cdot e^{-bt}\right\} = \varphi(p+b)$$

Different. im BILDRAUM:

$$\frac{d}{dp}\varphi(p) = \frac{d}{dp}\int_0^\infty f(t)\, e^{-pt}dt = -\int_0^\infty t\, f(t)\, e^{-pt}dt$$

$$\mathcal{L}\{t\, f(t)\} = -\frac{d\varphi(p)}{dp}$$

$$\mathcal{L}\, t^n f(t) = -\frac{d^n\varphi(p)}{dp^n},$$