$$\int_p^\rho \varphi(p)\,dp = \int_p^\rho \int_0^\infty f(t)\,e^{-pt}\,dt\,dp =$$

$$= \int_0^\infty f(t) \int_p^\rho e^{-pt}\,dp\,dt =$$

$$\rho \to \infty$$

$$= \int_0^\infty \frac{1}{t} f(t)\,dt$$

$$\mathcal{L}\,\frac{1}{t} f(t) = \int_p^\infty \varphi(p)\,dp$$

$$\int_0^\infty \frac{f(t)}{t}\,dt = \int_0^\infty \varphi(p)\,dp$$

$$\int_0^t \frac{f(t)}{t}\,dt = \int_0^\infty \frac{f(t)}{t}\,dt - \int_t^\infty \frac{f(t)}{t}\,dt$$

$$\frac{1}{p}\int_p^\infty \varphi(p)\,dp = \frac{1}{p}\int_0^\infty \varphi(p)\,dp - \mathcal{L}\int_t^\infty \frac{f(t)}{t}\,dt$$

$$\mathcal{L}\int_0^t F(t)\,dt = \frac{1}{p}\varphi^*(p). \quad \cancel{\mathcal{L}\int_0^t f(t)\,dt = \frac{1}{p}\varphi(p)}$$

$$F(t) = \frac{f(t)}{t}, \quad \cancel{\mathcal{L}^{-1}\varphi^*(p) = \frac{f(t)}{t}}$$

$$\mathcal{L}\,f(t) = \varphi(p) \qquad \mathcal{L}\,\frac{f(t)}{t} = \varphi^*(p)$$

$$\mathcal{L}\,\frac{f(t)}{t} = \int_p^\infty \varphi(p)\,dp = \varphi^*(p) = p\,\mathcal{L}\int_0^t \frac{f(t)}{t}\,dt$$

$$f(t) = 1 \qquad \mathcal{L}\,1 = \mathcal{L}\,E(t,0) =$$

$$= \int_0^\infty e^{-pt}\,dt = \frac{1}{p}$$

$$f(t) = e^{\alpha t}$$

$$\mathcal{L}\,e^{\alpha t} = \int_0^\infty e^{\alpha t} e^{-pt}\,dt =$$

$$R\alpha < Rp$$

$$= \int_0^\infty e^{-(p-\alpha)t}\,dt = \frac{1}{p-\alpha}$$

$$\mathcal{L}\,e^{\alpha t} = \frac{1}{p-\alpha}$$

$$\longrightarrow \quad \mathcal{L}\int_t^\infty \frac{f(t)}{t}\,dt = \frac{1}{p}\int_0^p \varphi(p)\,dp$$