

$$\varphi_1(p) \varphi_2(p) = \int_0^{\infty} e^{-pt} f_1(t) * f_2(t) dt$$

$$\mathcal{L}^{-1} \varphi_1(p) \varphi_2(p) = f_1 * f_2$$

$$\varphi_1(p) \varphi_2(p) = \mathcal{L} f_1 * f_2$$

$$m y'' + p y' + \kappa y = \varphi(t)$$

$$y(0) = 0$$

$$y'(0) = v_0$$

$$\mathcal{L} y' = p \bar{y}$$

$$\mathcal{L} y'' = p^2 \bar{y} - v_0$$

$$m(p^2 \bar{y} - v_0) + p p \bar{y} + \kappa \bar{y} = \bar{\varphi}$$

$$\bar{y} = \bar{\varphi}(p) \left\{ \frac{1}{[(p + \frac{p}{2m})^2 + \omega^2] m} + \frac{v_0}{(p + \frac{p}{2m})^2 + \omega^2} \right\}$$

$$\omega^2 = \frac{\kappa}{m} - \frac{p^2}{4m^2} > 0$$

$$y = \frac{1}{m\omega} \int_0^t \varphi(t-s) e^{-\frac{p}{2m}s} \sin \omega s ds + \frac{v_0}{\omega} e^{-\frac{p}{2m}t} \sin \omega t$$

$$= \frac{1}{m\omega} \int_0^t \varphi(s) e^{-\frac{p}{2m}(t-s)} \sin \omega(t-s) ds + \frac{v_0}{\omega} e^{-\frac{p}{2m}t} \sin \omega t$$